

# THREE-DIMENSIONAL TIME DOMAIN MODEL OF LIGHTNING INCLUDING CORONA EFFECTS\*

Andrew S. Podgorski

Electromagnetic Protection Group, Institute for Information Technology  
National Research Council of Canada, Ottawa, Canada K1A 0R6

## ABSTRACT

A new 3-D lightning model that incorporates the effect of corona is described for the first time. The new model is based on a Thin-Wire Time Domain Lightning (TWTDL) Code developed previously. The TWTDL Code was verified during the 1985 and 1986 lightning seasons by the measurements conducted at the 553-m CN Tower in Toronto, Ontario.

The inclusion of corona in the TWTDL code allowed study of the corona effects on the lightning current parameters and the associated electric field parameters.

## INTRODUCTION

To overcome problems resulting from the straight line channel approximation, the pre-defined channel current distribution, and the pre-defined channel current propagation speed, the author previously introduced the first fully 3-D time domain model of lightning based on the Thin-Wire Time Domain Code [1].

This new 3-D model not only accepted the 3-D geometry of the lightning channel but also calculated the lightning current distribution and speed of propagation of lightning current in the channel. In the model the provision was made for resistive and non-linear loading of the lightning channel, consequently permitting studies of the stepped leader and of the channel branching. The most important asset of the proposed model was the ability to model the effect of structures (towers, airplanes, etc.) on lightning current and vice versa. This led to the development of a unified lightning threat concept [2] that allowed definition of lightning parameters describing the lightning currents measured on the ground, towers, or airplanes [3,4,5].

Lightning that was modelled using proper resistance of the lightning channel demanded that the final breakdown point be located <20 m from the top of the tower. Considering this, it was always the

intention of the author to include the effects of corona in the proposed model of lightning, to account for measurements indicating that the location of the final breakdown point can be >100 m from the top of the tower. The corona model presented here was supported by a publication describing experiments on coronas generated in laboratories [6].

## THE THIN-WIRE TIME DOMAIN LIGHTNING MODEL

The Thin-Wire Time Domain Lightning model is based on a concept presented in Fig. 1 and it is obtained by combining the basic four models of positive, negative, upwards, and downwards lightning models of Berger [7] into one return stroke model. To form a new 3-D Thin Wire Time Domain Lightning Code (TWTDL), the Thin-Wire Time Domain (TWTD) Code and the Waterloo Analysis and Design (WATAND) Code are combined. The new TWTDL Code permits calculation of the currents of thin-wire structures using a moment method solution of the

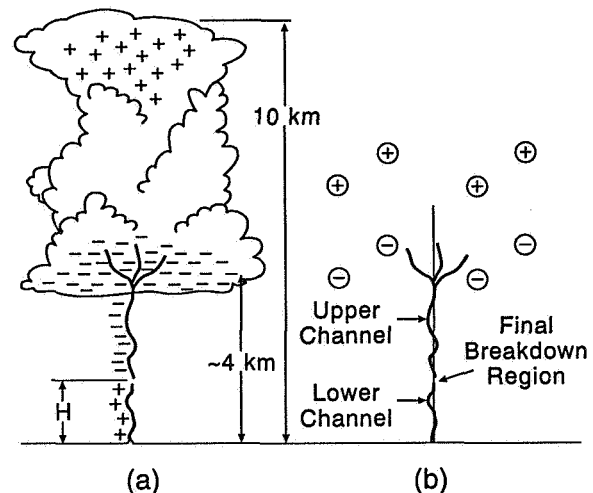


Figure 1. Return stroke phase of a lightning channel: (a) presented in Ref. 7; (b) used in the TWTDL model

\* NRC No. 31801

electric field Maxwell's integral equations. The Code computes a time domain solution by setting up a geometry-dependent matrix that relates the applied electric field to the induced currents and solves the matrix equation as an initial-value problem for the time-dependent induced current distribution. The induced currents are then used to find the time-dependent radiated and scattered fields. The thin-wire approximation used in the TWTDL Code is well suited for modelling of lightning since the diameter of the lightning channel is much smaller than its length.

The original TWTDL Code permitted modelling of only simple non-linear loads such as diodes. The addition of the Waterloo Analysis and Design (WATAND) computer code resulted in the ability of the TWTDL Code to accept resistances, capacitances, inductances, and non-linear and piecewise-linear voltage and current controlled resistances. The implementation of piecewise-linear resistances (switches) into the TWTDL Codes allowed the modelling of lightning branching and the implementation of voltage and current controlled resistances permitted modelling of non-linear effects during the attachment process. An example of the modelling of the effects of channel elongation process (stepped leader) on the lightning currents is shown in Fig. 2.

The TWTDL Code does not allow for DC charging of the lightning leader due to the requirement that the net charge on the modelled system always be zero. However, since the charge can be divided among different segments of the structure, the lightning channel charging can be accomplished by imposing a step charge at the cloud represented by multiplicity of short segments and by waiting long enough for stabilization of the initial charge and the field disturbance to occur (Fig. 3). Because of the 3-D character

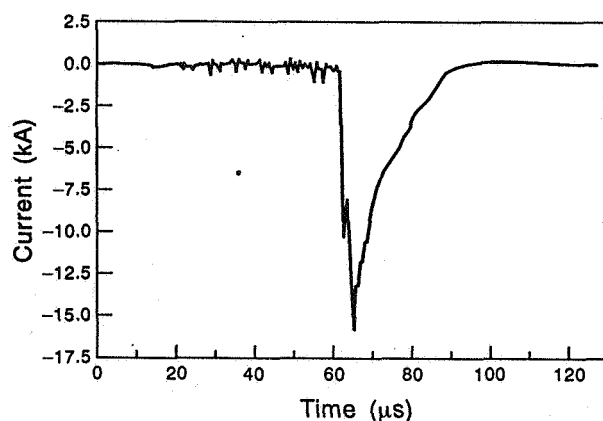


Figure 2. Time domain response of the lightning current that includes the effects of channel elongation process (stepped leader)

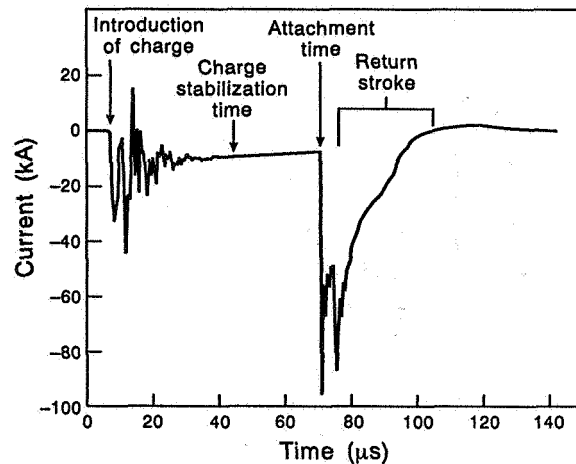


Figure 3. Charging mechanism in the TWTDL Code

of the TWTDL Code and its novel charging mechanism it was possible to model not only the cloud-to-ground lightning-tower interaction but also the intracloud and intercloud interaction of aircraft with lightning.

The validation of the TWTDL Code was accomplished through lightning current measurements conducted at the tallest free standing structure in the world, the 553-m CN Tower in Toronto, Canada. The CN Tower was chosen because of its free-standing character, dominating height, small overall diameter, and easy access to the location near the top of the tower where the free from the ground reflection lightning current measurements had to be made. The lightning current measurement system [8] installed at the CN Tower in 1985 had a response time of 50 ns. During the 1985-86 lightning seasons 94 lightning strokes were recorded. With the new measuring system, lightning current rise times in the order of 100 ns were recorded on tall towers for the first time. The measured rise time values compared very well with the values predicted by the TWTDL computer model. The same applies to the waveshape comparison, as can be seen on Fig. 4 where small details such as the effects of the attachment process, the ground reflection, the length of the lightning channel, and the height of the attachment region are correctly displayed. The comparison of measured and calculated waveforms revealed the presence of large numbers of measurements containing waveforms with very short 100 ns rise times.

The modelling of the 100 ns rise times required the placement of the final breakdown point at a distance of a few metres from the top at a tower, if the value of channel resistance was not to be decreased below acceptable levels. However, the presence of the final

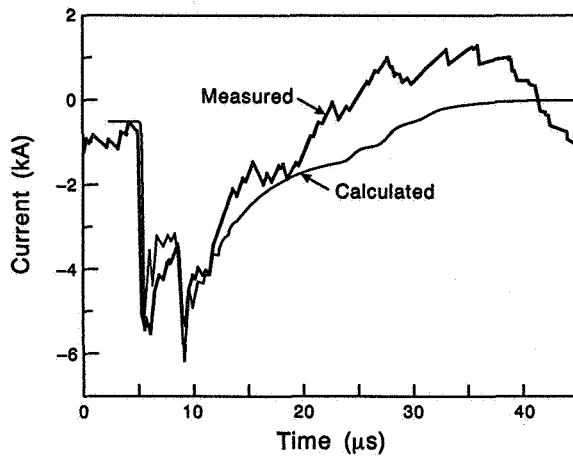


Figure 4. Calculated and measured lightning current waveform at the CN Tower

breakdown point so close to the top of the tower is not supported by video recordings. In the computer model, the distance of final breakdown point from the top of the tower could only be increased if there was a mechanism that could decrease the losses (resistance) and therefore the rise time of the current waveform. The only mechanism that could be responsible for such a decrease of risetime could have been the presence of corona. The description of cold and relatively dark corona around a lightning channel given in Ref. 6 allows the inclusion of the corona effects into the previous lightning model based on the use of the TWTDL Code.

### MODELLING OF CORONA EFFECTS

Reference 6 stipulates the presence of corona not only during the interstroke interval but in all phases of the lightning discharge. The corona charge is deposited around the thin lightning channel by a radial electric field pushing it away from the channel.

In relatively large electric fields existing near the channel, the radial electric field will carry the corona charge away at the speed of light. Reference 6 suggests that the radius of lightning corona expands up to 120 m and implies that a corona envelope of this size is relatively dark and not easily observable.

In order to model the effects of radial corona, radially resistively loaded wires were added to the previous lightning model. Figure 5 shows the structural geometry of the new lightning model. In this model the lightning, corona channels, and the CN Tower are described in terms of 3-D straight wire segments loaded with resistances. The segment length rule for the TWTDL Code is defined by:

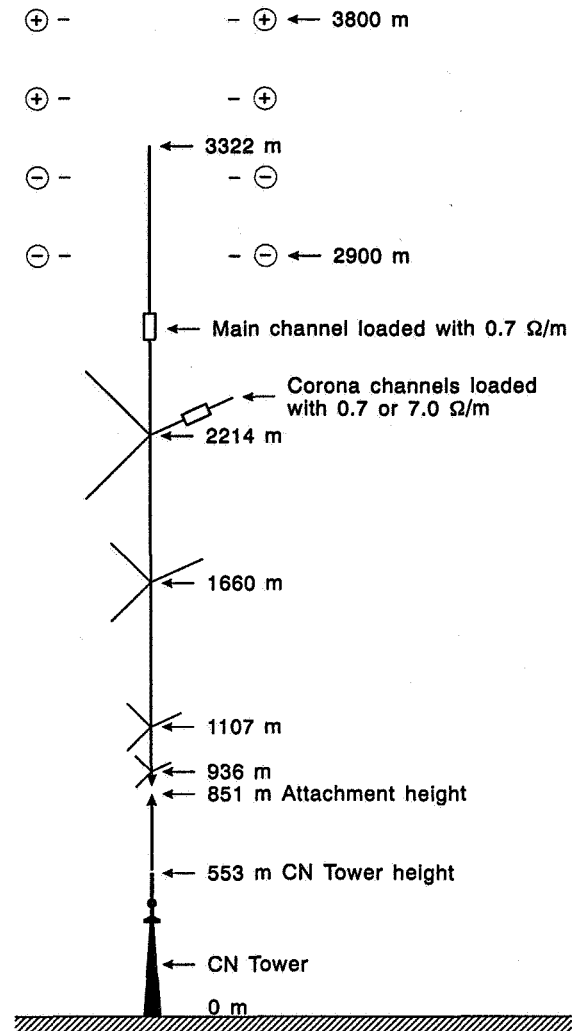


Figure 5. A 3-D view of a TWTDL input structure simulating corona effects

$$L \leq c \Delta t \quad (1)$$

where  $\Delta t$  is the duration of the time step and  $c$  is the velocity of light in vacuum.

Since it was the author's intention to analyze the lightning behavior using a time resolution comparable to the shortest rise time reported, a time step of a 140 ns was used. For 140 ns time step the model required 78 segments to model the main lightning channel and an additional 120 segments to model the effects of corona.

The thin-wire approximation used in the TWTDL Code required that the segment diameter be less than the segment length. While the exact limits have not

been determined, the following has been found to give good results:

$$D \leq 1.2L \quad (2)$$

where  $D$  is the segment diameter and  $L$  the segment length. The attachment region of the model Fig. 5 was modelled by a non-linear (voltage or current controlled) resistor, with an OFF resistance of  $10 \Omega$  and ON resistance of  $3 \Omega$ , series inductor and parallel capacitor.

The resistive loading of the lightning channel was determined from the experiments conducted at the CN Tower. A resistance value of  $30 \Omega$  for a 42-m segment ( $0.7 \Omega \text{ m}^{-1}$ ) was used, as this value was found to give the best results when the measured and calculated waveshapes of analyzed lightning were compared. The resistive loading of the corona channel was varied between  $0.7$  and  $7 \Omega \text{ m}^{-1}$ .

## STUDY OF LIGHTNING CURRENT

In the study of corona modelling it was assumed that the lightning channel is vertical and straight. This assumption was not required by the model but it greatly simplified the analysis. To account for an average lightning stroke response the height of the attachment region was placed at a height of 277 m over the top of the 553-m tall CN Tower. The height was chosen on the basis of the current rise time that for such a height has an average value of about 500 ns for models that either include or do not include corona. Figure 6 presents the values of the peak current amplitude of the lightning current as a

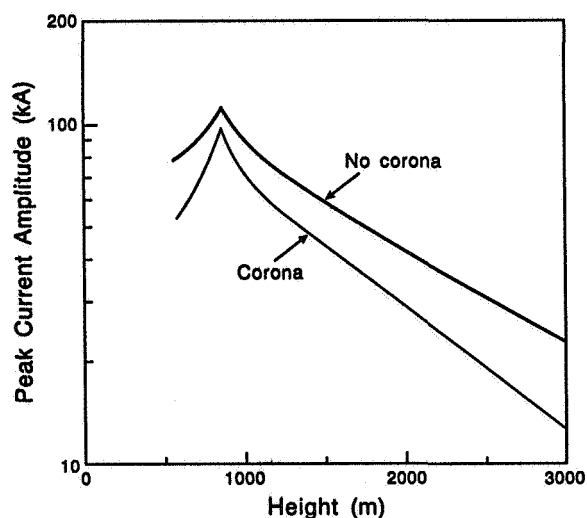


Figure 6. The peak current amplitude of the lightning current as a function of the position along the lightning channel

function of the position along the lightning channel. The changes are drastic and indicate introduction of losses into propagation along the lightning channel. The corona peak current reduction is smallest at the attachment point and equals 25% of the peak current without the corona. At a height of 2000 m above the attachment point the effect of corona results in a 40% reduction in a peak current amplitude.

The rise time of the lightning current waveform along the lightning channel is displayed by Fig. 7. It can be seen that, in the regions on both sides of the attachment point, a 30% reduction of the current rise time occurs. The region of the rise time reduction extends 200 m up and 200 m down from the attachment point. In regions further away from the attachment point the presence of corona increases the rise time of the lightning current waveform. However, this increase does not exceed 20% up to a height of 2 km above the attachment point.

The effect of corona on the rise time of the lightning current explains the presence of 100 ns rise times measured at the CN Tower [2]. Figure 7 shows that without the presence of corona the variation of the height of the attachment point with respect to the top of the tower cannot be larger than 20 m in order to provide for the rise times shorter than 340 ns. The presence of corona extends this variation of the height of the attachment point to 100 m. The presence of the attachment region at a height of 100 m above the top of the CN Tower is easily supported by the video recordings.

The increase of rise time as a result of the presence of corona explains the presence of high peak current

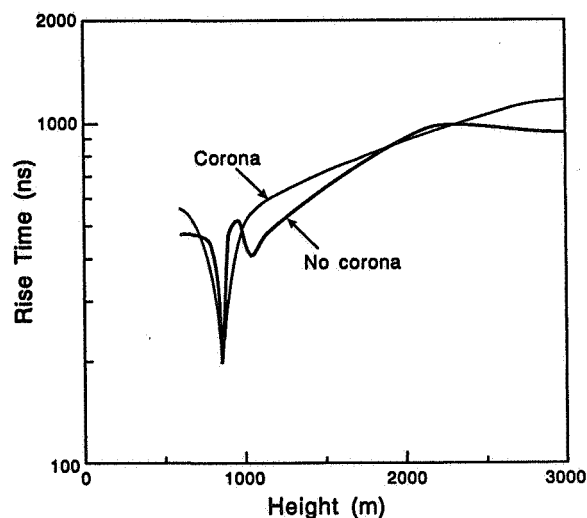


Figure 7. The rise time of the lightning current as a function of the position along the lightning channel

derivatives measured at towers and airplanes [4]. Figure 8 indicates that, in proximity to the attachment region, the presence of corona extends the region of high peak current derivatives. It is interesting to note that outside of the attachment region the corona reduces the peak current derivative by about 40%.

One of the parameters that created much controversy in the past is the velocity of current wavefront in the lightning channel. Figure 9 shows the velocity of the current wavefront normalized to the speed of light. It should be noted that this calculation was made for a straight lightning channel. Therefore, assuming channel tortuosity of 50%, one can easily divide the normalized velocity numbers of Fig. 9 by a factor of two and obtain average velocity in order of 40 to 45% of the velocity of light.

The important conclusion from Fig. 9 is, however, related to the variation of the velocity of propagation of low and high frequency components of the propagating current wavefront. Figure 9 displays the velocity of propagation of two points at the front of the current waveform. One of these points is located in the middle (50%) of the waveform and the other at its peak. The 50% point can be related to the high frequency components of the waveform, while the peak point can be related to the low frequency components of the waveform. Figure 9 reveals that the high frequency components (50% point) are propagating with the velocity close to the velocity of light, while the low frequency components travel with velocity considerably lower than the velocity of

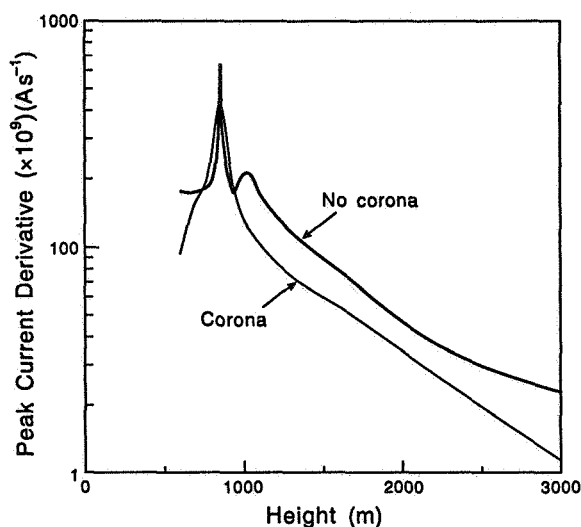


Figure 8. The peak current derivative as a function of the position along the lightning channel

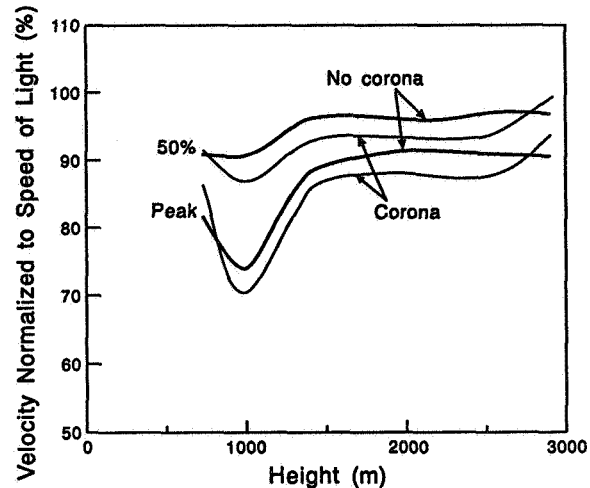


Figure 9. Velocity at propagation of the lightning wavefront as a function of the position along the lightning channel

light. The slowest velocity of propagation occurs in the attachment point region.

It is interesting to note the very small effect of corona on the velocity of propagation of both the low and high frequency components of the current waveform. The corona decreased the velocity of propagation of the two waveforms by only 5%. However, the presence of corona has a substantial effect on the electric field perpendicular to the surface of the lightning channel, as shown in Fig. 10. This field responsible for propagation of corona is reduced by the presence of corona channels. Figure 10 indicates that, in cases without corona, the region with the electric field higher than 1 MV/m extends up to a height of 600 m above the attachment point. With the presence of corona the field of 1 MV/m extends only to a height of 200 m above the attachment point. It appears that the corona is self-confining.

This finding is better displayed in Fig. 11 where a comparison of charge density of lightning channel with corona (charge  $Q_c$ ) and without corona (charge  $Q_{nc}$ ) is presented. Figure 11 shows that up to the height of 800 m above the attachment region the charge of the lightning channel is increased by 15% in the presence of corona.

## STUDY OF RADIATED FIELDS

In the TWTDL Code, the values of the radiated electric fields (E-fields) are calculated from previously determined lightning current values. However, a perfectly conducting ground is assumed for the purpose of calculation of the E-fields generated

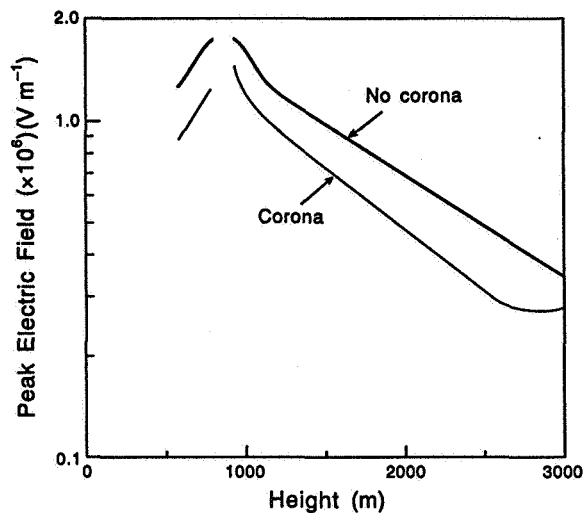


Figure 10. Peak of the electric field perpendicular to the lightning channel as a function of the position along the channel

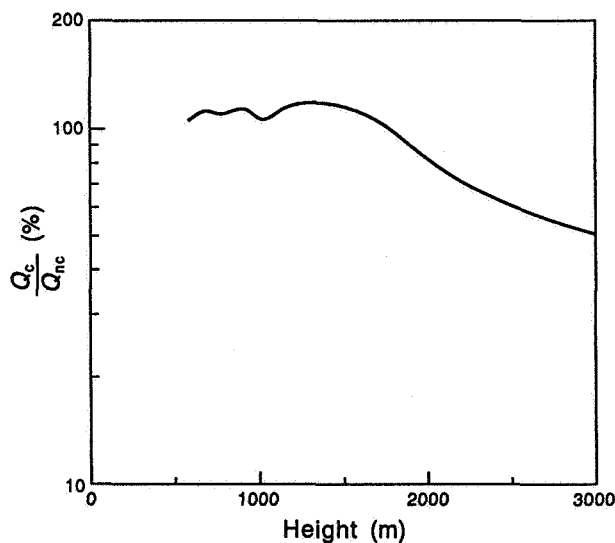


Figure 11. Comparison of charge density of lightning channel with corona ( $Q_c$ ) and without corona ( $Q_{nc}$ )

by the lightning channel. This simplification allows the total field to be calculated by summarizing the effects of previously calculated lightning currents and their underground images. The approximation used gives very good results for high frequency radiated components of the E-field, but it may create some problems for middle frequency components as these components can be trapped in a wave propagating parallel to the ground. The perfect grounding assumption permits accurate prediction of the

E-fields of the lightning channel located over the sea water, a distance of a few kilometres from the shore.

The TWTDL Code permits the verification of the relationship between lightning current and electric fields parameters. The peak E-field amplitude calculated as a function of distance from lightning channel using the TWTDL Code is shown in Fig. 12. From Fig. 12 it should be noted that the E-field peak amplitude decreases with the inclusion of corona. It can be seen that the low resistance corona, modelled with a corona channel resistance of  $0.7 \Omega \text{ m}^{-1}$ , is responsible for much greater field reduction than the corona modelled with a corona channel resistance of  $7 \Omega \text{ m}^{-1}$ . Corona appears to be responsible also for the narrowing of the near field region. It can be seen that the near field region is reduced from 5 km, for a case where there is no corona, to about 1 km, if the effect of corona is included. A very interesting phenomena can be observed at a distance less than 1 km from the lightning channel; for the high resistance corona the E-field values are higher than the E-field calculated when no corona is considered. With the use of Figs. 6 and 12 one can easily establish the relationship between the peak current amplitude and the peak amplitude of E-field. One can show that the coefficient defining the relationship between the peak current amplitude and the peak amplitude of the E-field varies up to  $\pm 20\%$  totally, for the near and the far field, for the case with or without corona, and for different heights of the attachment point. One should realize, therefore, that these large errors will result in substantial errors if peak current levels are determined from the E-field measurements. This approach, however, is used

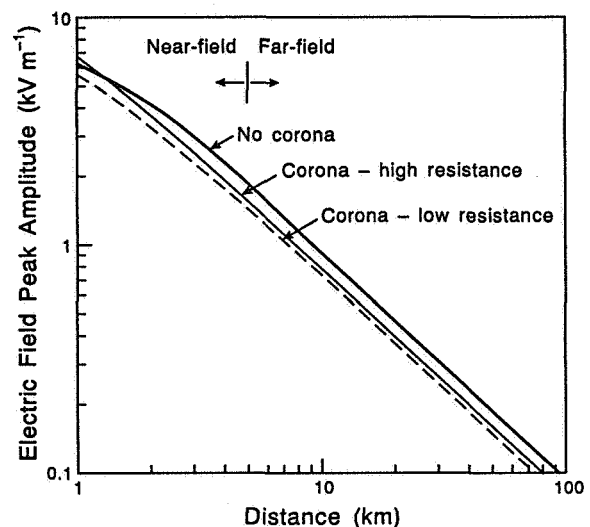


Figure 12. The peak E-field amplitude as a function of the distance from the lightning channel

when establishing a significant lightning current data base (10 million strokes) is required. Assuming  $\pm 20\%$  error, one can estimate that the peak current amplitude may be contained anywhere between 20 and 30 kA. Consequently, for high accuracy of peak current amplitude data, only direct measurements of the lightning currents should be considered.

On the contrary, the estimating of lightning current rise time through the measurements of the rise time of the E-fields results in exceptionally small error. However, the measurements of the rise time of E-field should be conducted at a distance not greater than 1 km from the lightning, channel as displayed in Fig. 13. Comparison between Fig. 7 and Fig. 13 indicates that the lightning current rise time at the attachment point and the rise time of the peak E-field are closely related. Slightly slower rise time of the peak E-field results from the corona shielding effect of the radiated field that is being measured on the ground.

### CONCLUSIONS

One of the important challenges faced by researchers working in the area of lightning is the measurement of lightning parameters. The direct lightning measurements provide researchers with a limited data set, such as peak current amplitude, rise time, and peak current derivatives. It is obvious that, if the data set is to be expanded into the low probability of lightning occurrence region, one will have to estimate the lightning current parameters from the electromagnetic field generated by lightning. The TWTDL code provides an excellent tool since using a 3-D analysis of electromagnetic field allows very accurate

derivation of the relationship between the lightning current and radiated electromagnetic field. The inclusion of the effect of corona further expands the capabilities of the Code and makes it the most comprehensive tool for modelling the lightning interaction with structures.

### REFERENCES

- [1] Podgorski, A.S., and Landt, J.A., "Three Dimensional Time Domain Modelling of Lightning", *IEEE Transactions on Power Delivery*, Vol. PWRD-2, No. 3, pp 931-938, July 1987.
- [2] Podgorski, A.S., "A Case for a Unified Lightning Threat", *Proceedings of the International Aerospace and Ground Conference on Lightning and Static Electricity*, Oklahoma City, OK, pp. 118-122, April 19-22, 1988.
- [3] Podgorski, A.S. and Podgorski, E.M., "Lightning Standards for Protection of Ground Electronic Systems", *Proceedings of the International Aerospace and Ground Conference on Lightning and Static Electricity*, Oklahoma City, OK, pp. 324-327, April 19-22, 1988.
- [4] Podgorski, A.S., "Lightning Standards for Aircraft Protection", *Proceedings of the 1990 IEEE International Symposium on Electromagnetic Compatibility*, Washington, DC, pp. 218-223, August 21-23, 1990.
- [5] Podgorski, A.S., "The Low Probability Lightning Threat.", *Proceedings of the EMC EXPO - 91 Conference*, Orlando, FL, June 25-27, 1991. In press.
- [6] Heckman, S.J., and Williams, E.R., "Corona Envelopes and Lightning Currents", *Journal of Geophysical Research*, Vol. 94, No. D11, pp. 13287-13294, September 30, 1989.
- [7] Berger, K., "Lightning", *Physics of Lightning*, R.H. Golde, Ed. Academic Press, Vol. 1, pp. 119-190, 1977.
- [8] Podgorski, A.S., "A New System for Measurement of Lightning Currents on Towers", *Proceedings of the IEEE Instruments and Measurement Technology Conference*, Boulder, CO, pp. 71-73, March 25-27, 1986.

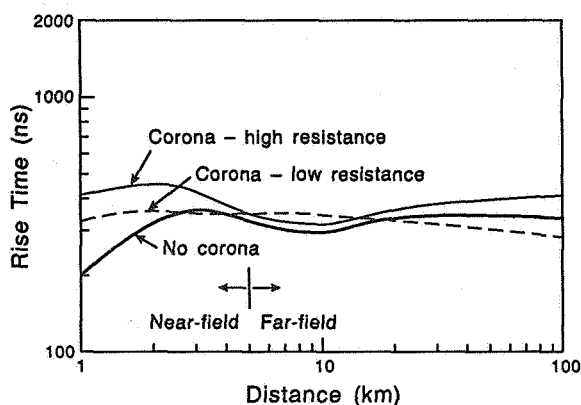


Figure 13. The rise time of the E-field as a function of the distance from the lightning channel